DIELECTRIC FUNCTION OF A DEGENERATE ELECTRON GAS IN THE PRESENCE OF A STEADY MAGNETIC FIELD

P. MISRA

DEPARTMENT OF PHYSICS, RAVENSHAW COLLEGE, CUTTACK

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ABSTRACT. Using Boltzmann—Vlassov equation we derive expressions for the frequency and wave-number dependent dielectric function of a degenerate electron gas in the presence of a steady magnetic field which are valid at and near absolute zero temperatures

INTRODUCTION

In the presence of a magnetic field, the electron gas becomes anisotropic, the wavenumber and frequency dependent Response function $\tilde{K}(\vec{k}, w)$ of Lindhard is then a tensor. When the wave number vector is parallel to the direction of the external magnetic field, $\tilde{K}_{11}(\vec{k}, \omega) = \tilde{K}_{22}(k, \omega)$, $\tilde{K}_{12}(\vec{k}, \omega) = -\tilde{K}_{21}(\vec{k}, \omega)$ and all other components except these and $\tilde{K}_{33}(\vec{k}, \omega)$ are identically zero. The last mentioned component is independent of the magnetic field strength The aim of the present investigations is to obtain explicit expressions for $\tilde{K}_{11}(\vec{k}, \omega)$ and $\tilde{K}_{12}(\vec{k}, \omega)$ for a degenerate electron gas at and near absolute zero temeprature when the external magnetic field is steady and uniform. The dynamics of the electron gas is described by the Boltzmann- Vlassov equation. The equilibrium distribution function is a Fermi-Dirac distribution The prescription for integration across the singularity encountered in the expressions for $\tilde{K}_{\alpha\beta}(\vec{k}, \omega)$ are obtained by invoking causality (Pradhan 1962) as illustrated in our previous work (Misra 1962). For evaluation of integrals over Fermi-Dirac distribution function near absolute zero temperature Sommerfeld's (1928) approximate method has been used.

DIELECTRIC FUNCTION FROM VLASSOV EQATION

The wave-number and frequency dependent dielectric function is given by

$$\epsilon_{\alpha\beta}(k, \omega) = 1 + \frac{4\pi i\tilde{K}_{\alpha\beta}(k, \omega)}{\;} \tag{1}$$

where
$$\tilde{J}_{\alpha}(\bar{k}, \omega) = \tilde{K}_{\alpha\beta}(k', \omega)E_{\alpha\beta}(k', \omega) \tag{2}$$

The current density $J(\vec{r}, t)$ resulting from fluctuating electric field $\vec{E}(\vec{r}, t)$ in a plasma is given by

$$J_\alpha(\vec{r}, t) = -e \int d^3u \, u_\alpha f_1(\vec{r}, \vec{u}, t) \qquad \ldots \quad (3)$$

where $f_1(\vec{r}, \vec{u}, t)$ obeys the linearised Boltzmann-Vlassov equation

$$\frac{\partial f_1}{\partial t} + \vec{u}\cdot\nabla_r f_1 - \frac{e}{mC}(\vec{U}\times\vec{B})\cdot\vec{\nabla}_u f_1 = \frac{ne}{m}\vec{E}(\vec{r}, t)\cdot\vec{\nabla}_u f_0 \qquad \ldots \quad (4)$$

whose space-time Fourier transform, in a cylindrical co-ordinate system (u, v, θ) for $\vec{u}$ with $\vec{B}$ and $\vec{k}$ along Z axis, is given by

$$i(ku-\omega)f_1(\vec{k}, \vec{u}, \omega) + \Omega \frac{\partial f_1(\vec{k}, \vec{u}, \omega)}{\partial \theta} = \frac{ne}{m}\left[\frac{\partial f_0}{\partial v}(E_1\cos\theta + E_2\sin\theta) + E_3\frac{\partial f_0}{\partial u}\right] \qquad \ldots \quad (5)$$

where $$\Omega = \frac{eB}{mC}$$

The solution of this equation is immediately seen to be

$$f_1(\vec{k}, \vec{u}, \omega) = \frac{ne}{m}\left[\frac{iku\cos\theta - i\omega\cos\theta + \Omega\sin\theta}{(ku-\omega)^2 - \Omega^2}\left(\frac{\partial f_0}{\partial v}\right)E_1 + \frac{iku\sin\theta - i\omega\sin\theta - \Omega\cos\theta}{(ku-\omega)^2 - \Omega^2}\left(\frac{\partial f_0}{\partial v}\right)E_2 + \frac{i}{ku-\omega}\left(\frac{\partial f_0}{\partial u}\right)E_3\right] \qquad \ldots \quad (6)$$

and which when used in equations (1), (2) and (3) give us

$$\epsilon_{11}(\vec{k}, \omega) = 1 - \frac{4\pi\omega_0^2}{\omega}\int_{-\infty}^{+\infty} du \frac{(ku-\omega)F(u)}{(ku-\omega)^2 - \Omega^2} \qquad \ldots \quad (7a)$$

$$\epsilon_{12}(\vec{k}, \omega) = 1 - \frac{4\pi\omega_0^2}{\omega}\int_{-\infty}^{+\infty} du \frac{\Omega F(u)}{(ku-\omega)^2 - \Omega^2} \qquad \ldots \quad (7b)$$

$$\epsilon_{33}(\vec{k}, \omega) = 1 + \frac{4\pi\omega_0^2}{\omega}\int_{-\infty}^{+\infty} du \frac{u\frac{\partial F(n)}{\partial u}}{ku-\omega} \qquad \ldots \quad (7c)$$

where $\omega_0{}^2 = \dfrac{4\pi ne^2}{m}$ and $F(u) = \dfrac{1}{4}\int_0^\infty v^2 dv \dfrac{\partial f_0}{\partial v} = -\dfrac{1}{2}\int_0^\infty dv\, vf_0$

It will be noticed that the integrands in the above expressions are singular at $u = \dfrac{\omega+\Omega}{k}$, $u = \dfrac{\omega-\Omega}{k}$ and $u = \dfrac{\omega}{k}$. The prescription for integration across these poles obtained from causality is to add a small positive imaginary part to ω in the integrands.

COMPUTATION OF $\widetilde{K}_{11}$ AND $\widetilde{K}_{12}$ FOR A FERMI DISTRIBUTION

For a Fermi distribution

$$nf_0 = 2\left(\frac{m}{h}\right)^3 \frac{1}{e^{-\nu + \frac{1}{2}m\beta(u^2+v^2)}+1}$$

and hence

$$F(u) = \frac{1}{4}\int_0^\infty v^2 dv \frac{\partial f_0}{\partial v} = -\frac{1}{nm\beta}\left(\frac{m}{h}\right)^3 \log\left(1+e^{\nu-\frac{1}{2}m\beta u^2}\right)$$

Therefore from equations (1) and (7a)

$$\tilde{k}_{11}(\vec{k}, \omega) = -\frac{i\omega_0{}^2}{2}\left[\int_{-\infty}^{+\infty} du \frac{F(u)}{\omega-\Omega-ku+i\epsilon} + \int_{-\infty}^{+\infty} du \frac{F(u)}{\omega+\Omega-ku+i\epsilon}\right]$$

$$= \frac{i\omega_0{}^2}{2nk}\left(\frac{m}{h}\right)^3 \int_0^\infty du\, u \frac{\log\dfrac{(\omega-\Omega+ku)(\omega+\Omega+ku)}{(\omega-\Omega-ku)(\omega+\Omega-ku)}}{1+e^{-\nu+\frac{1}{2}m\beta u^2}}$$

$$+\frac{\omega^2{}_0}{2knm\beta}\left(\frac{m}{h}\right)^3 \log\left[1+e^{\nu-\frac{m\beta}{2k^2}(\omega-\Omega)^2}\right]\left[1+e^{\nu-\frac{\beta m}{2k^2}(\omega+\Omega)^2}\right] \quad \ldots \quad (8)$$

The integration over u cannot be analytically carried out. However for temperatures near absolute zero approximate analytical forms can be obtained by using Sommerfeld's method of integration as is done in the work of Pradhan and Misra (1960) and Misra and Misra (1962). We shall not go into the details but give the final results :

$$Im\,\widetilde{K}_{11}(\vec{k},\,\omega) = \frac{\omega_0^2}{2nk}\left(\frac{m}{h}\right)^3\left[2v_0^2\omega/k+\frac{1}{2}\log\frac{(\omega-\Omega)(\omega+\Omega)+2v_0k\omega+v_0^2k^2}{(\omega-\Omega)(\omega+\Omega)-2v_0k\omega+v_0^2k^2}-\right.$$

$$\left.-\frac{1}{2}\frac{(\omega-\Omega)^2}{k^2}\log\frac{\omega-\Omega+v_0k}{\omega-\Omega-v_0k}-\frac{1}{2}\frac{(\omega+\Omega)^2}{k^2}\log\frac{\omega+\Omega+v_0k}{\omega+\Omega-v_0k}\right]$$

$$+\frac{\pi^2\omega_0^2}{24m^2\beta^2v_0^2nk}\left(\frac{m}{h}\right)^3\log\frac{(\omega-\Omega)(\omega+\Omega)+2v_0k\omega+v_0^2k^2}{(\omega-\Omega)(\omega+\Omega)-2v_0k\omega+v_0^2k^2}$$

$$+\frac{\pi^2\omega_0^2}{6m^2\beta^2n\,v_0}\left(\frac{m}{h}\right)^3\left[\frac{\omega-\Omega}{(\omega-\Omega)^2-v_0^2k^2}+\frac{\omega+\Omega}{(\omega+\Omega)^2-v_0^2k^2}\right] \quad \ldots \quad (9a)$$

$$Re\,\widetilde{K}_{11}(\vec{k},\,\omega) = \frac{\omega_0^2}{8k}\left\{\frac{2\pi}{n}\left(\frac{m}{h}\right)^3\left[v_0^2-\frac{(\omega-\Omega)^2}{k^2}\right]\right.$$

$$\left.+\frac{\pi^2}{16m^2\beta^2v_0^5}\right\}\theta\left[v_0^2-\frac{(\omega-\Omega)^2}{k^2}\right]+\frac{\omega_0^2}{8k}\left\{\frac{2\pi}{n}\left(\frac{m}{h}\right)^3\left[v_0^2-\frac{(\omega+\Omega)^2}{k^2}\right]\right.$$

$$\left.+\frac{\pi^2}{16m^2\beta^2v_0^5}\right\}\theta\left[v_0^2-\frac{(\omega+\Omega)^2}{k^2}\right] \quad \ldots \quad (9b)$$

For the computation of $\widetilde{K}_{12}(\vec{k},\,\omega)$ we proceed in a similar manner and finally obtain

$$Re\,\widetilde{K}_{12}(\vec{k},\,\omega) = \frac{-\omega_0^2}{2nk}\left(\frac{m}{h}\right)^3\left[\frac{2v_0\Omega}{k}+\frac{1}{2}v_0^2\log\frac{(\omega+\Omega)(\omega-\Omega)-2v_0k\Omega-v_0^2k^2}{(\omega+\Omega)(\omega-\Omega)+2v_0^2k\Omega-v_0k^2}\right.$$

$$\left.-\frac{(\omega+\Omega)^2}{2k^2}\log\frac{\omega+\Omega+v_0k}{\omega+\Omega-v_0k}+\frac{(\omega-\Omega)^2}{2k^2}\log\frac{\omega-\Omega+v_0k}{\omega-\Omega-v_0k}\right]$$

$$-\frac{\omega_0^2\pi^2}{24m^2\beta^2v_0^2nk}\left(\frac{m}{h}\right)^3\log\frac{(\omega+\Omega)(\omega-\Omega)-2v_0k\Omega-v_0^2k^2}{(\omega+\Omega)(\omega-\Omega)+2v_0k\Omega-v_0^2k^2}$$

$$-\frac{\pi^2\omega_0^2}{6m^2\beta^2nv_0}\left(\frac{m}{h}\right)^3\left[\frac{\omega+\Omega}{(\omega+\Omega)^2-v_0^2k^2}-\frac{\omega-\Omega}{(\omega-\Omega)^2-v_0^2k^2}\right] \quad \ldots \quad (10a)$$

$$Im\,\widetilde{K}_{12}(\vec{k},\,\omega) = -\frac{\omega_0^2}{8k}\left\{\frac{2\pi}{n}\left(\frac{m}{h}\right)^3\left[v_0^2-\frac{(\omega+\Omega)^2}{k^2}\right]\right.$$

$$\left.+\frac{\pi^2}{16m^2\beta^2v_0^5}\right\}\theta\left[v_0^2-\frac{(\omega+\Omega)^2}{k^2}\right]+\frac{\omega_0^2}{8k}\left\{\frac{2\pi}{n}\left(\frac{m}{h}\right)^3\left[v_0^2-\frac{(\omega-\Omega)^2}{k^2}\right]\right.$$

$$\left.+\frac{\pi^2}{16m^2\beta^2v_0^5}\right\}\theta\left[v_0^2-\frac{(\omega-\Omega)^2}{k^2}\right] \quad \ldots \quad (10b)$$

In the limit of vanishing B, the expression for $\widetilde{K}_{11}(\vec{k},\,\omega)$ coincides with the expression for the same obtained in an earlier paper for the magnetic field free case and the $\widetilde{K}_{12}(\vec{k},\,\omega)$ vanishes identically.

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